Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY
Summer Examination-2017

Subject Name : Topology

Subject Code : 4SC06TOC1 **Branch :B.Sc. (Mathematics)**

Semester : 6 **Date :19/04/2017** **Time: 2:30 To 5:30** **Marks :70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define: Indexed set. (01)

b) Define: Trivial topology. (01)

c) Give an example of countable set and uncountable set? (01)

d) True/false: Intersection of two topologies is also topology. (01)

e) Define : s-topology (01)

f) Give an examples of open set in $\mathbb{R}$ with usual topology. (01)

g) Define: Left ray –topology on $\mathbb{R}$. (01)

h) True/false: Every closed set of topological space contains all of it's limit points. (01)

i) Define: Compact space. (01)

j) Define: T_0– space. (01)

k) Write the closer of the set A= $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\ldots..\}$. (01)

l) What do you mean by dense set? (01)

m) True/false: $\bar{\bar{A}} = \bar{A}$. (01)

n) State finite intersection property. (01)

Attempt any four questions from Q-2 to Q-8.

Q-2 **Attempt all questions** **(14)**

a) Define: Topological space.Check whether the following collections are topology on$X$. **(06)**

(1) $X = \{a, b\}$ and $\tau = \{\emptyset, \{a\}, X\}$
(2) $X = \{1, 2, 3\}$ and $\tau = \{\emptyset, \{1\}, \{2\}, \{1,2\}, X\}$

b) Define Co – finite topology. If X is nonempty set then prove that show that (X, τ) is topological space where τ is co –finite topology on X. **(05)**

c) If X is any nonempty set then show that P(X) is topology on X. What is the name of such a topology? **(03)**

Q-3 **Attempt all questions** **(14)**

a) If $X = \mathbb{R}$ then show that the collection **(06)**

$\tau = \{U \subseteq \mathbb{R} \mid x \in U \exists \varepsilon > 0 \ni (x-\varepsilon, x+\varepsilon) \subseteq U\} \cup \{\emptyset\}$ is topology on $\mathbb{R}$.

b) What do you mean by aclosed set? Prove that finite union of closed set is closed. **(05)**

c) Which of the following is open, closed or neither open nor closed set ? **(03)**

(1) (1,2)

(2) {1,2}

(3) $\mathbb{R} - \mathbb{Q}$

Q-4 **Attempt all questions** **(14)**

a) Define: Disconnected set. If (X, τ) is topological space then prove that X is disconnected iff $\exists A \subseteq X \ni A$ is both open and closed. **(06)**

b) Define: Door space. If X={a , b , c } and τ ={ $\emptyset$, X , { b} , {a , b } , { b , c } }. Show that (X, τ) is door space. **(05)**

c) If $X = N$ then prove that **(03)**

$\tau = \{ U \subseteq N / U = \{ m, m+1, m+2, .. \}, m \in N \} \cup \{\emptyset\}$ is topology on N

Q-5 **Attempt all questions** **(14)**

a) Define: Closer of set. Prove that **(07)**

(i) $\bar{A}$ is smallest closed set containing A.

(ii) A is closed if and only if $\bar{A} = A$.

b) Prove that $\overline{A \cup B} = \bar{A} \cup \bar{B}$. Is it true that $\overline{A \cap B} = \bar{A} \cap \bar{B}$? Justify your answer. **(07)**

Q-6 **Attempt all questions** **(14)**

a) Define: Homeomorphism. Prove that homeomorphism is an equivalence relation. **(07)**

b) Define: Continuous function. If X = { a , b , c } , τ_1 = { $\emptyset$, X,{a} , {b, c} } and Y = {1, 2, 3, 4} , τ_2 = { $\emptyset$, Y ,{4} , { 1 , 2 , 3 } } also define a map $f: (X, \tau_1) \to (Y, \tau_2)$ Such that f(a) = 4 ,f(b) = 2 ,f(c) = 3 then show that f is continuous function. **(05)**

c) What is hereditary property? Is openness of subset hereditary property of topological space? **(02)**

Q-7 **Attempt all questions** **(14)**

a) Prove that any closed subset of a compact space is compact. **(06)**

b) Prove that continuous image of a compact space is compact. **(05)**

c) If (X, τ) and (X, τ^*) are two topological spaces such that τ^* is finer than τ and (X, τ) is a T_2 space then prove that (X, τ^*) is T_2 space. **(03)**

Q-8 **Attempt all questions** **(14)**

a) State and prove Heine - Borel theorem. **(07)**

b) Prove that every subspace of T_2 space is T_2space. **(04)**

c) Show that R is T_1space with usual topology. **(03)**